Sem - 4

04/05/2020.

Theory of testing of hypothesis

After estimation of parameters by statistics now we test whether the selected sample is good fitted to the sample or not

To perform testing of sample, we use various test statistics →

(1) Standard Normal distribution

$$Z = \frac{x - \mu}{\sigma} \sim N(0, 1)$$

and also follows all properties of normal distribution.

(2) Z or Tau (τ) distribution

$$\tau = \frac{\bar{x} - \mu}{SE(\bar{x})} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

and it also follows the properties of normal distribution

(3) Students 't' distribution

$$t = \frac{\bar{x} - \mu}{SE(\bar{x})} \sim t(n-1)$$

$$= \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$ where s = Sample s.d

$(n-1)$ is degrees of freedom.

(2) and (3) are used to test the population parameter i.e. population mean: (μ).

Null hypothesis (H_0)

First we set a hypothetical value of this parameter i.e initially we assume a given value of μ.

$$H_0 : \mu = \mu_0$$

Alternative hypothesis (H_1)

Apart from null hypothesis, the other two possibilities are either $\mu > \mu_0$ or $\mu < \mu_0$. This is in turn written as,

$$H_1 : \mu \neq \mu_0 \text{ or}$$

$$H_1 : \mu \gtrless \mu_0$$

We use two confidence level i.e.

(a) 95 % level of confidence => 5 % level of significance

(b) 99 % level of confidence => 1 % level of significance

Test of Population Mean (μ)

Case 1: When σ^2 is known and sample size is large and small

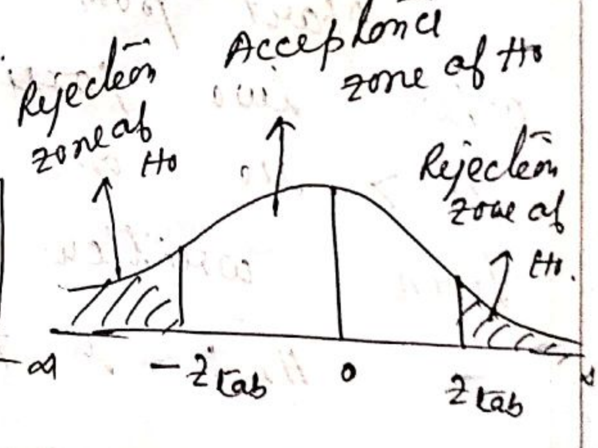
When population variance (σ^2) is known then we use Z or Tau distribution to test population mean (μ). The sample size may be large or small whatever.

The value of test statistic is,

$$Z = \frac{\bar{x} - \mu_{H_0}}{SE(\bar{x})}$$

$$= \left[\frac{\bar{x} - \mu_{H_0}}{\sigma/\sqrt{n}}\right]$$

$$= \pm Z_{cal.}$$



We take, $|Z| = Z_{cal.}$

The hypothesis are, $H_0 : \mu = \mu_0$

$H_1 : \mu \gtrless \mu_0$

This is both tailed test

At 95% level of confidence or 5% level of significance the tabulated value of Z is,

$|Z_{tab}| = 1.96$ in case of both tailed

$|Z_{tab}| = \pm 1.645$ in case of single tailed test

When $-Z_{tab} < Z_{cal} < Z_{tab}$ then we accept the H_0 i.e the sample has regarded from the population whose mean is μ_0.

Therefore, when $|Z_{cal}| < |Z|_{tab}$ then H_0 is accepted and when $|Z_{cal}| > |-Z_{tab}|$ then H_0 is rejected

$\therefore$ $|Z_{cal}| > 1.96$ then H_0 is rejected at 5% significance level.

$|Z_{cal}| < 1.96$ then H_0 is accepted at 5% significance level.

Similarly in case of 1 % level of significance or 99 % confidence interval,

$|z_{tab}| = 2.58$ in case of both tailed

$|z_{tab}| = \pm 2.33$ in case of single tailed

<u>Confidence limit for μ</u>

(a) 95 % confidence limit of μ is, $\mu = \bar{x} \pm 1.96 \, \sigma/\sqrt{n}$

(b) 99 % confidence limit of μ is

$$\mu = \bar{x} \pm 2.58 \, \sigma/\sqrt{n}$$

<u>Case 2:</u> <u>When σ^2 is unknown and sample size is small ($n<30$)</u>

When σ^2 is unknown and sample size is small ($n<30$) we use Student's 't' distribution to test population mean (μ).

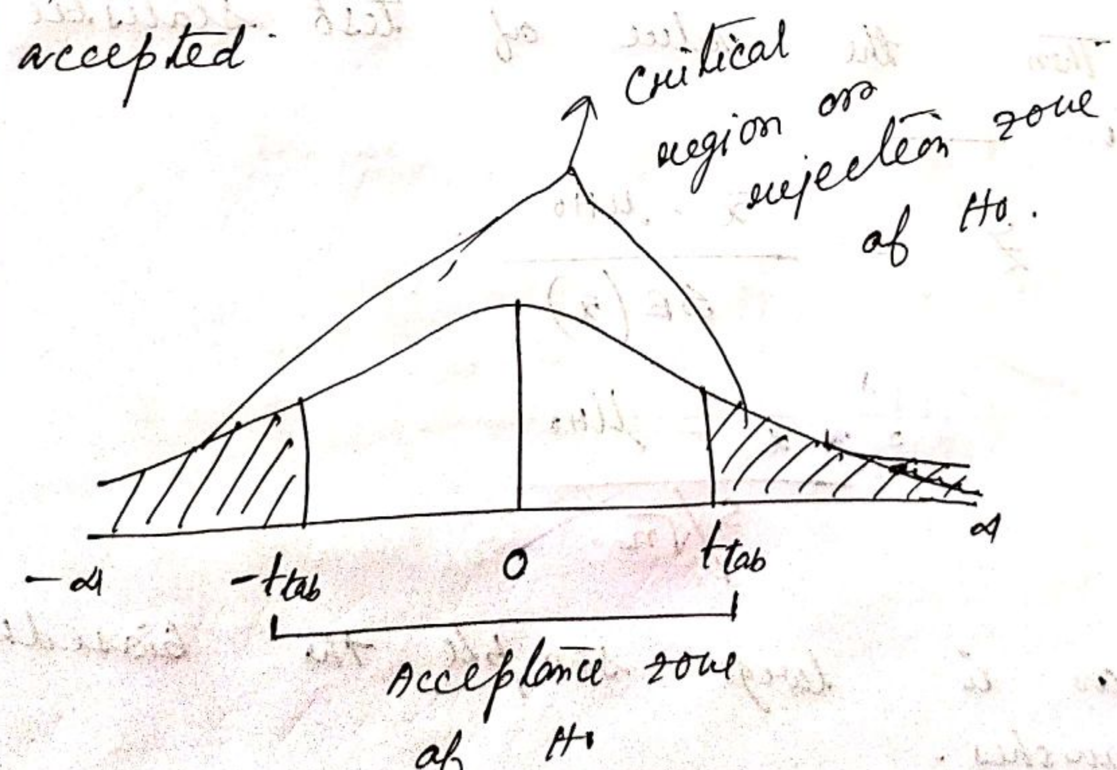
The value of test statistic is $\longrightarrow$

$$t = \frac{\bar{x} - \mu_{H_0}}{SE(\bar{x})} \sim t_{(n-1)}$$

$$= \frac{\bar{x} - \mu_{H_0}}{s/\sqrt{n-1}}.$$

As sample variance is a biased estimator of population variance. then we put $\frac{\sigma}{\sqrt{n}} = s/\sqrt{n-1}$.

As before. if $|t_{cal}| > |t_{tab}|_{(n-1)}$ at 5% or 1% level of significance then H_0 is rejected and if $|t_{cal}| < |t_{tab}|_{n-1}$ then H_0 is accepted.



Confidence limit of μ

(a) 95 % confidence limit of μ

$$\Rightarrow \bar{x} \pm t_{0.025} \, s/\sqrt{n-1}$$

(b) 99 % confidence limit of μ

$$\Rightarrow \bar{x} \pm t_{0.005} \, s/\sqrt{n-1}.$$

Case : 3 : When σ^2 is unknown and sample size is large $(n>30)$

When σ^2 is unknown and sample size is large $(n>30)$ then we use Z distribution with sample variance

Then the value of test statistic is,

$$Z = \frac{\bar{x} - \mu_{H_0}}{SE(\bar{x})}$$

$$= \frac{\bar{x} - \mu_{H_0}}{s/\sqrt{n}}$$

as in large sample the biasedness vanishes.

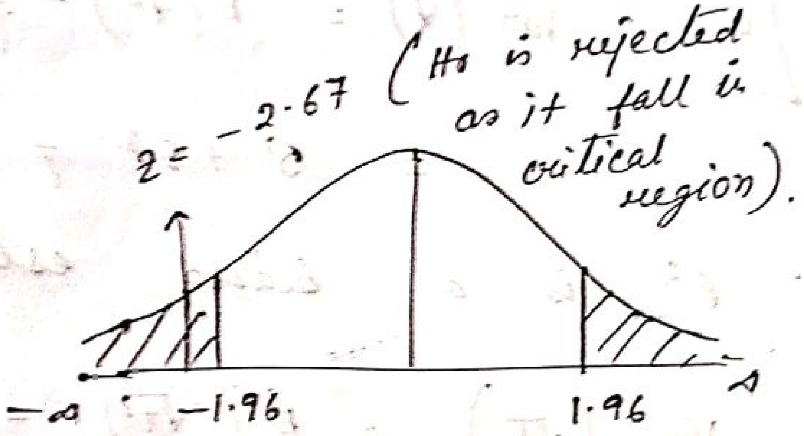
$$\sigma^2 = \left(\frac{n}{n-1}\right) \cdot s^2$$

$$= s^2 \left(\frac{1}{1 - 1/n}\right)$$

when $n \rightarrow \infty$ then $1/n \rightarrow 0$

$$\therefore \boxed{\sigma^2 = s^2}$$

56 Graph =)



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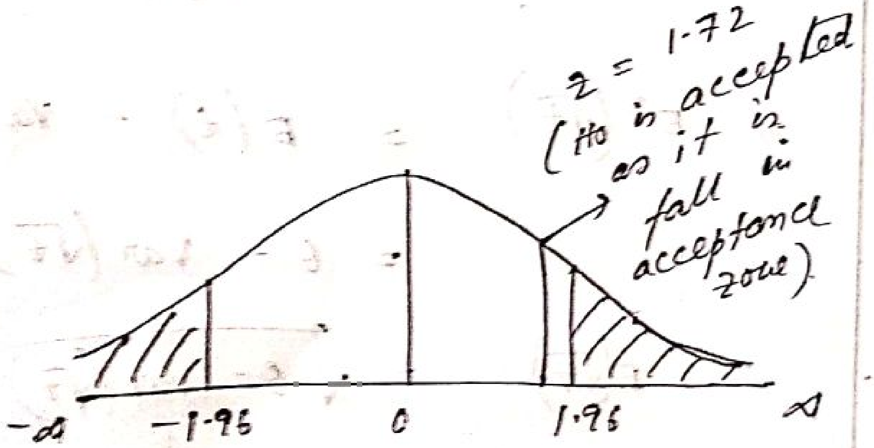

Estimation

4 Since t is an unbiased estimator of θ then,

$$E(t) = \theta$$

Now $E(t^2) = Var(t) + [E(t)]^2$

[as $Var(t) = E(t^2) - \{E(t)\}^2$]

$$= \sigma^2 + \theta^2 \neq \theta^2$$

$\therefore$ t^2 is a biased estimator of θ^2

$$Var(\sqrt{t}) = E(\sqrt{t}^2) - [E(\sqrt{t})]^2$$

$$= E(t) - E^2(\sqrt{t})$$

$$\Rightarrow E^2(\sqrt{t}) = E(t) - Var(\sqrt{t})$$

$$= \theta - Var(\sqrt{t})$$

$$\Rightarrow E(\sqrt{t}) = \sqrt{\theta - Var(\sqrt{t})} \neq \sqrt{\theta}$$

$\therefore$ $\sqrt{t}$ is a biased estimator of $\sqrt{\theta}$